

Precession of Spinning Tops

11-24-2018 Tohei Moritani

1. Objectives Measurement of revolution rates of spinning tops and their precession. Comparison with theory.

2. Theory The angular velocity of the precession ω_p is obtained as follows:

$$\omega_p = \frac{mgh}{I \omega} \quad (1)$$

ω_p : The angular velocity of the precession [rad/s]

m : The mass of a top [kg]

g : Gravitational acceleration rate [m/s²]

h : The height from a support to the center of gravity [m]

I : The moment of inertia [kg·m²]

ω : Angular velocity of a top [rad/s]

回転角速度 ω と回転数 f は次の関係がある。

$$\omega = 2\pi f$$

式(1)は次のように書き換えられる。

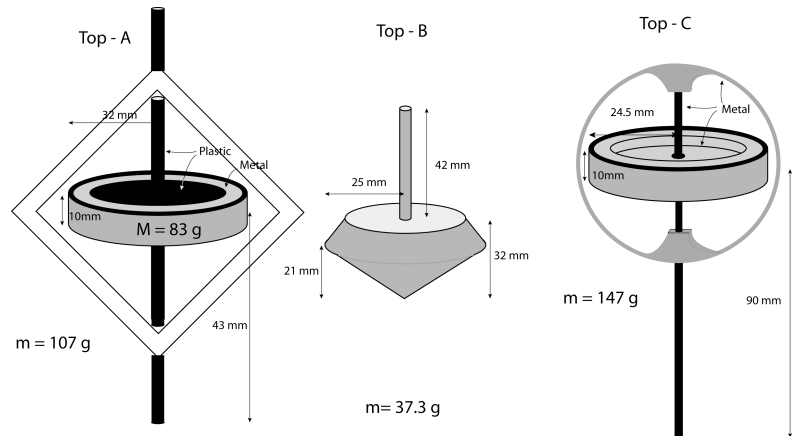
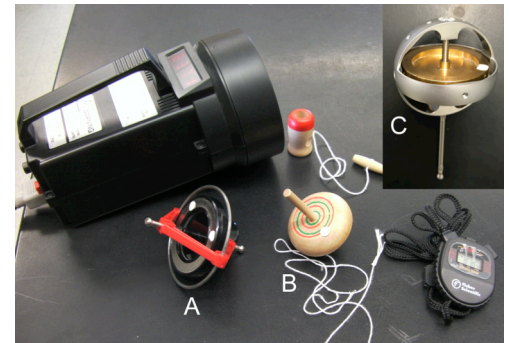
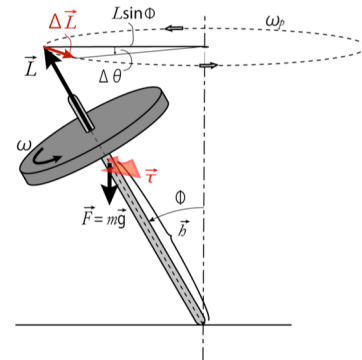
$$f_p = \frac{mgh}{4\pi^2 I f} \quad (2)$$

f_p : Revolution rate of precession [Hz]

f : Revolution rate of a top [Hz]

3. Experiment

- 1) Three kinds of tops: A, B and C
- 2) Revolution of a top is measured with a stroboscope.



4. Results

		Top - A	Top - B	Top - C	
I		MR ² M=83g	½ MR ²	½ mR ²	mR ²
	kg·m ²	8.18 x 10 ⁻⁵	1.17 x 10 ⁻⁵	4.21 x 10 ⁻⁵	8.42 x 10 ⁻⁵
m	kg	107 x 10 ⁻³	37.3 x 10 ⁻³	147 x 10 ⁻³	
h	m	43 x 10 ⁻³	21 x 10 ⁻³	90 x 10 ⁻³	
f (obs)	rpm	1910	836	5251	
f (obs)	Hz	31.8	13.9	87.5	
f _p (obs)	Hz	0.41	0.80	0.61	
f _p (calc)	Hz	0.44	1.2	0.85	0.43

5. Discussion

There is ambiguity about the calculation of the moment of inertia. In Top-A, f_p (obs) and f_p (calc) agree within 7% on the model of a hoop ($I = MR^2$). In Top-C, f_p (obs) shows the intermediate value between two values of f_p (calc) calculation based on the two models, hoop ($I = MR^2$) and disk ($I = \frac{1}{2} MR^2$).